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Recently, three experiments have been proposed in order to show that the standard and Bohmian quantum mechanics can have different predictions at the individual level of particles. However, these thought experiments have encountered some objections. In this work, it is our purpose to show that our basic conclusions about those experiments are still intact.

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## I. INTRODUCTION

The standard quantum mechanics (SQM) involves a set of rules that allow physicists to evaluate statistical correlations between data associated with the experimental procedures of preparation and measurement, using the wave function of a system of particles. In fact, the statistical interpretation of the wave function is in accord with all experiments that have been performed yet. However, Bohm [1] in 1952 proposed his subquantum realistic theory which is now often called Bohmian quantum mechanics (BQM), with a more detailed description than SQM, so that it deals directly with the properties of quantum objects rather than with merely statistical results. In other words, BQM provides a realistic interpretation of quantum phenomena by adding hidden variables, representing the position of particles, to the wave function. Hence, all particles have well-defined positions at all times, and follow trajectories determined by the quantum wave function  $\psi$  using the guidance condition

$$\dot{\mathbf{x}}_i = \frac{\hbar}{m_i} \text{Im} \left( \frac{\nabla_i \psi}{\psi} \right), \quad (1)$$

which its unitary time development is governed by Schrödinger's equation.

Bohm's theory outlined above constitutes a consistent theory of motion about which more details can be found in [2]. By the way, in order to ensure the compatibility of the motion of an ensemble of particles with the results of SQM, Bohm [1] put the further constraint

$$P = |\psi|^2 \quad (2)$$

on the density of the probability  $P$ , a constraint which is sometimes called the quantum equilibrium hypothesis (QEH). It should be noted that, QEH is a consistent subsidiary condition imposed on the causal theory of motion and has no more fundamental status than that.

However, if we deal with individual systems, we can hope that the determination of one of the initial positions of the system, using an ingenious way, provides grounds of differentiation between SQM and BQM, although because of QEH, it is evident that they certainly give the same statistical prediction for an ensemble of the individual particles. Concerning this point, recently Ghose [3] and these authors [4-6] proposed some experiments which could differentiate SQM from BQM. But, Marchildon [7] accompanied by Struyve and De Baere [8] have claimed that the proposed experiments cannot provide different predictions for SQM and BQM. Although Ghose [9,10] and these authors [5,6] believe that Marchildon's objections do not change our main results, Marchildon still holds that our constraint on  $y_0$  in [5,6] is doubtful [11].

In this work, we have presented a concise review of our three experiments and a more detailed discussion on them in order to make firm our previous basic conclusions, particularly against the objections raised in [7,8].

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In order to have a more complete examination on the three suggested experiments in [3-6], we have presented a summary of each experiment separately, as well as our reasons against the objections raised in [7,8].

### A. A two-particle double-slit experiment with an entangled wave function

Consider a double-slit screen with two identical slits  $A$  and  $B$  with the width  $2\sigma_0$  and centers located at  $(0, \pm Y)$ , in the  $x - y$  plane. Instead of the usual one particle emitting source, consider a special point source emitting pairs of identical non-relativistic entangled particles, and which is located very far from the two-slit screen. The entanglement of particles 1 and 2 is described by the following conditions:

$$\begin{aligned} y_1 + y_2 &= 0, \\ p_{1y} - p_{2y} &= 0. \end{aligned} \quad (3)$$

The general wave function of this two-particle system, after diffraction from the two slits, is given by [3,5]

$$\psi(x_1, y_1; x_2, y_2; t) = N[\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t) \pm \psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t)]. \quad (4)$$

To have a complete discussion, we assume that the two slits produce the Gaussian wave packets along the  $y$ -direction at  $t = 0$ , that is,

$$\psi_{A,B}(x, y, t) = (2\pi\sigma_t^2)^{-1/4} e^{[-(\pm y - Y - u_y t)^2 / 4\sigma_0\sigma_t + ik_y(\pm y - Y - u_y t/2)]} e^{i(k_x x - E_x t/\hbar)}, \quad (5)$$

where

$$\sigma_t = \sigma_0 \left(1 + \frac{i\hbar t}{2m\sigma_0^2}\right), \quad (6)$$

and

$$u_y = \frac{\hbar k_y}{m}, \quad E_x = \frac{1}{2} m u_x^2. \quad (7)$$

Based on SQM, it is well known that the probability of simultaneous detection of the pair of particles, at arbitrary points  $y_1 = Q_1$  and  $y_2 = Q_2$  on the screen, is

$$P_{12}(Q_1, Q_2, t) = \int_{Q_1}^{Q_1+\Delta} \int_{Q_2}^{Q_2+\Delta} dy_1 dy_2 |\psi(y_1, y_2, t)|^2, \quad (8)$$

where  $\Delta$  represents the width of a particle detector on the screen.

On the other hand, using BQM, we obtained the equation of motion for the  $y$ -coordinate of the center of mass of the two particles in ref. [5] as

$$y(t) = y_0 \sqrt{1 + (\hbar t / 2m\sigma_0^2)^2}, \quad (9)$$

where  $y = (y_1 + y_2)/2$ , and  $y_0$  is the vertical coordinate of the center of mass at  $t = 0$ . It is clear that, if the condition  $y_0 = 0$  is considered, then the center of mass will remain on the  $x$ -axis for all times. Thus, based on BQM, each entangled pair of particles will be always detected symmetrically with respect to the  $x$ -axis. However, SQM predicts that the probability of asymmetrical detection of the pairs of particles can be different from zero, at variance with BQM's symmetrical prediction. Furthermore, according to SQM's prediction, the probability of finding two particles at one side of the  $x$ -axis can be non-zero, while it is shown that BQM forbids such events, provided that  $y_0 = 0$ .

Here, somebody may feel reluctant to use the  $y_0 = 0$  constraint (for example, [7,8]). In fact, one may argue that  $y_0$  must be distributed according to  $|\psi|^2$ , because of QEH. We agree that the properties of the individual particles, that is,  $y_1$ ,  $y_2$ ,  $p_{1y}$  and  $p_{2y}$ , are undetermined based on QEH, but it should be noted that their joint properties are completely defined, as shown on page 77 of ref. [12] and in the following. In fact, although the operators  $\hat{y}$  and  $\hat{p}_y$  do not commute for each particle,

$$[\hat{y}_i, \hat{p}_{iy}] = i\hbar, \quad (10)$$

but the operators for  $(\hat{y}_1 + \hat{y}_2)$  and  $(\hat{p}_{1y} - \hat{p}_{2y})$  do commute, i.e.,

$$[(\hat{y}_1 + \hat{y}_2), (\hat{p}_{1y} - \hat{p}_{2y})] = 0. \quad (11)$$

Therefore, for the entangled wave function, the joint properties  $(y_1 + y_2)$  and  $(p_{1y} - p_{2y})$  could both be determined with an arbitrary accuracy. In addition, using this conclusion, the objection of Struyve and De Baere [8] about the non-ergodic property of BQM, which is one of Ghose's arguments about the incompatibility of SQM and the conventional de Brogli-Bohm theory [10,11], cannot be sustained.

However, we also studied a case in which we can have  $\Delta y_0 \neq 0$  and  $\langle y_0 \rangle = 0$ . It is shown that, for obtaining symmetrical detection with a reasonable approximation it is enough to require the following constraint [5]

$$0 \leq y_0(t) \ll \sigma_0, \quad (12)$$

where  $y_0(t)$  shows variation of  $y_0$  with time. Once again, due to the entanglement of the particles, it is a possible constraint, although each particle has its quantum equilibrium distribution, that is,

$$(\Delta y_1)_{t=0} = (\Delta y_2)_{t=0} \sim \sigma_0. \quad (13)$$

In fact, contrary to the single-particle two-slit experiment which always requires the constraint  $\Delta y_0 \sim \sigma_0$ , because of the uncertainty on both particle's position and momentum, the case of two-entangled particle double-slit experiment has the advantage of  $y_0$  determination with the required accuracy and so, for example, we do not need to take very small slits in order to make sure that the particles depart from the  $x$ -axis symmetrically, as was offered by Struyve and De Baere [8].

On the other hand, concerning the  $y_0$  determination, Marchildon [11] believes that once particles have gone through the slits, much of their memory of coming from the source is erased. He has argued that, if the wave packets coming out of the slits are as in eq. (5) at  $t = 0$ , then the  $y$ -coordinates are spread independently, with the standard deviations of the order of  $\sigma_0$ , and the center of mass coordinate  $y$  has a similar spread. To answer this kind of criticism, we should investigate the issue of entanglement of particles, using both SQM and BQM.

Using SQM, one can think of the entanglement of the two particles at the source, in the form of the two following alternatives:

1. The entanglement constraint  $(y_1 + y_2)_{t \rightarrow -\infty} = 0$  at the source, is erased during the diffraction of the pair at the slits at  $t = 0$ . Thus, the results of the joint probability in eq. (8) becomes inconsistent with BQM's symmetrical prediction which still maintains the entanglement property of the two particles. The validity of the entanglement of the two particles in BQM after the diffraction, has been further discussed in the following.
2. SQM agrees that the entanglement property is maintained at all times, i.e.  $y_1(t) + y_2(t) = 0$ , because there is no interaction between the pair and the double-slit screen and, all conditions are identical for the two particles. Then, the joint probability density is given by

$$P_{12}(y_1(t), -y_1(t), t) = |\psi(y_1(t), -y_1(t), t)|^2. \quad (14)$$

But, there is a problem here. It is clear that, the joint probability density

$$P_{12}(y_1(t), y_2(t), t) = |\psi(y_1(t), y_2(t), t)|^2 \quad \text{if : } y_1(t) + y_2(t) \neq 0, \quad (15)$$

has a non-zero value. In fact,  $P_{12}(y_1(t), -y_1(t), t)$  cannot be equal to 1 for every entangled pair, as was mentioned by Struyve and De Baere [8], if one uses SQM. However, these authors use SQM to refute our statements about BQM. Thus, we should accept that either the asymmetrical detection of the two particles with erased entanglement is a right prediction or that SQM is an incomplete theory. But, accepting the first alternative means inconsistency between SQM and BQM, as we have shown in the following.

It is well known in BQM that, the wave function  $\psi$  plays two conceptually different roles:

1. As determining the influence of the environment on the particle via the guidance condition (1).
2. As determining the probability density  $P = |\psi|^2$ , i.e. QEH.

But, the primary conceptual role for  $\psi$  in Bohm's theory is the first role. In fact, in BQM, probability only enters as a subsidiary condition on a causal theory of the motion of individual particles in order to make statistical prediction equivalent with SQM, i.e., the statistical meaning of the wave function is a secondary property. In our experiment too, BQM requires that the entanglement of the two particles must be considered in their tracks, so that the quantum distribution of the pairs on the screen is formed according to the equation of motion

$$y_1(t) + y_2(t) = 2y_0 \sqrt{1 + (\hbar t / 2m\sigma_0^2)^2}, \quad (16)$$

which is weighted by the probability density  $P = |\psi|^2$ . By considering the equation of motion for the center of mass coordinate  $y$ , we shall have the time development of the entanglement property of the two particles which originated from the source at  $t \rightarrow -\infty$ . Hence, in BQM, rather than SQM, the entanglement of the two particles is not forgotten. By the way, it is shown that the final interference pattern is the same as the one predicted by SQM [3,5], which shows the consistency of the result obtained for the ensemble of particles with QEH. It is worthy to note that, in Bohm's theory, underlying quantum mechanics is a causal theory of the motion of waves and particles, which is consistent with a probabilistic interpretation, but does not require it. Therefore, contrary to the case of SQM, in BQM, which is a causal theory, the entanglement is not erased and in consequence, SQM's probabilistic prediction must differ from BQM's deterministic prediction, at the individual level, although the wave function used for the two theories as well as their final predicted interference patterns are the same.

Although we have studied the important objections to GGA experiment [3,5], it is useful to have a more detailed discussion about some of the remaining objections. Marchildon [7] as well as Struyve and De baere [8] claim that for  $\hbar t / 2m\sigma_0^2 \ll 1$ , SQM also predicts symmetrical detection as BQM does. Although we agree that under this condition the predictions of the two theories approaches the same results, we still hold that SQM's and BQM's results for this experiment are really different, because Gaussian slits with the width  $2\sigma_0$  and the distance  $2Y \sim \sigma_0$  produces a small but finite overlapping of the two wave functions in the domain  $|y| < Y$  on the screen which prevents symmetrical prediction, using SQM. In addition, to make the subject clearer, we can consider either  $\sigma_0 \rightarrow 0$  and  $m \rightarrow \infty$  or any other desired variations that could produce

$$\frac{\hbar t}{2m\sigma_0^2} \sim 1. \quad (17)$$

This condition yields  $y \sim y_0$  due to eq. (9). By considering  $Y \sim \sigma_0$ , eq. (17) means that there is a considerable overlap of the two wave packets emerging from the two slits on the screen and, in consequence, SQM does not predict symmetrical detection. However, BQM can predict acceptable symmetrical detection if the constraint

$$0 \leq \Delta y_0 \ll \sigma_0 \quad (18)$$

is applied, which is feasible for the two entangled particles. Thus, our previous basic conclusion in refs. [5,6] is still unchanged, when this condition holds.

On the other hand, Struyve and De Baere [8] stated that, if, e.g.,  $\sigma_0$  is considered very small so that  $\hbar t / 2m\sigma_0^2 \gg 1$ , then an asymmetrical detection on the screen is predicted by BQM, although  $y_0$  was considered very small and we assured that the particles depart the  $x$ -axis symmetrically. Clearly, we agree with this special case. But, we have shown that for the two entangled particles,  $y_0$  is adjustable at the source with a desired precision and we do not need to take  $\sigma_0$  very small to make sure of symmetrical departure of the two particles from the  $x$ -axis. In addition, for the case of a double-slit experiment with two unentangled particles, we have shown that, even this special case can lead to a different prediction between SQM and BQM, using selective detection [4,5]. We have considered the discussion of this experiment in the next subsection.

### B. A two-particle double-slit experiment with an unentangled wave function

In this experiment [4,5], we have considered a special form of the last experimental set-up, and our source is substituted with another one that emits two unentangled identical particles. Hence, the wave function of this two-particle system is given by

$$\psi(x_1, y_1; x_2, y_2; t) = \overline{N}[\psi_A(x_1, y_1, t) + \psi_B(x_1, y_1, t)][\psi_A(x_2, y_2, t) + \psi_B(x_2, y_2, t)], \quad (19)$$

where  $\psi_A$  and  $\psi_B$  can be considered as the Gaussian wave packets (5). SQM's probabilistic prediction about the joint detection of the two particles on the screen is the same as the one noted for the last experiment, i.e. equation (8).

On the other hand, according to BQM, the velocity of the center of mass of the two particles in the  $y$ -direction is [4,5]

$$\dot{y} = \frac{(\hbar / 2m\sigma_0^2)^2 y t}{1 + (\hbar / 2m\sigma_0^2)^2 t^2} + \overline{N} \frac{\hbar}{2m} \text{Im} \left\{ \frac{1}{\psi} \left( \frac{Y + u_y t}{\sigma_0 \sigma_t} + 2ik_y \right) (\psi_{A_1} \psi_{A_2} - \psi_{B_1} \psi_{B_2}) \right\}, \quad (20)$$

so that the equation of motion for the  $y$ -coordinate of the center of mass can be written as

$$y(t) \simeq y_0 \sqrt{1 + (\hbar t / 2m\sigma_0^2)^2}, \quad (21)$$

if the conditions  $Y \ll \sigma_0$  and  $k_y \simeq 0$  are satisfied. Furthermore, the guidance condition (1) yields

$$\begin{aligned} \dot{y}_1(x_1, y_1; t) &= -\dot{y}_1(x_1, -y_1; t), \\ \dot{y}_2(x_2, y_2; t) &= -\dot{y}_2(x_2, -y_2; t), \end{aligned} \quad (22)$$

which imply the  $y$ -component of the velocity of each particle must vanish on the  $x$ -axis, independent of the other particle's position. If the case  $y_0 = 0$  were possible, then one could have reconsidered the last discussion on SQM's probable asymmetrical prediction and BQM's symmetrical one. But, as it is well known, we agree that for this two-unentangled particle experiment, we have

$$\Delta y_0 \sim \sigma_0, \quad (23)$$

according to QEH. However, we still can obtain different predictions for the two theories. To show this, consider the case of  $\Delta y_0 \sim \sigma_0$  and  $\langle y_0 \rangle = 0$ . We assume that to obtain symmetrical detection around the  $x$ -axis with reasonable approximation, it is enough that the center of mass variation be smaller than the distance between any two neighboring maxima on the screen. Then, we have [5]

$$\Delta y_0 \ll \frac{\pi \hbar t}{Ym}, \quad (24)$$

which yields

$$Y \ll 2\pi\sigma_0, \quad (25)$$

where in (24) the condition  $\hbar t / 2m\sigma_0^2 \sim 1$  is assumed, so that we have  $y \sim y_0$ . Thus, for obtaining symmetrical detection in BQM, we should guarantee that the conditions  $Y \ll 2\pi\sigma_0$  and  $\hbar t / 2m\sigma_0^2 \sim 1$  be satisfied for this experiment. It should be noted that, under these conditions, the two wave packets are overlapped on the screen in an interval of the order of  $\sigma_0$ . In this interval neither BQM nor SQM predict symmetrical detection around the  $x$ -axis. In fact, the symmetrical detection predicted by BQM happens at far (relative to  $\sigma_0$ ) from the  $x$ -axis on the screen. In other words, save the central peak, which does not show symmetry with respect to the  $x$ -axis, other less prominent maxima appear at the locations

$$\begin{aligned} y_{n+} &\simeq n_+ \frac{\pi \hbar t}{Ym} \pm \Delta y, \\ y_{n-} &\simeq -n_- \frac{\pi \hbar t}{Ym} \pm \Delta y, \end{aligned} \quad (26)$$

where  $y_{n\pm}$  refer to  $y$ -component of the maxima above or below the  $x$ -axis on the screen, respectively. In addition,  $n_{\pm}$  represent positive integer numbers. BQM's symmetrical prediction puts the following constraint:

$$n_+ = n_-. \quad (27)$$

However, SQM's probabilistic prediction does not require this constraint. Note that, under the condition  $Y \ll \sigma_0$ , we have

$$\frac{\pi \hbar t}{Ym} \gg \Delta y, \quad (28)$$

because for the condition  $\hbar t / 2m\sigma_0^2 \sim 1$ , we have  $\Delta y \sim \Delta y_0 \sim \sigma_0$ . That is, in BQM, we have symmetrical peaks up to  $\sigma_0$ . It is clear that, the wave function (19) predicts the same interference patterns for the ensemble of particles for the two theories, according to QEH. By the way, the same discussion on the determination of the conditions for the case of  $\Delta y_0 \sim \sigma_0$  can be applied as well to the case of a double-slit experiment with two entangled particles.

Furthermore, for the case of  $\hbar t / 2m\sigma_0^2 \gg 1$ , using eqs. (21) and (22) and selective detection of the two particles, which requires registration of those two particles that are detected at the two sides of the  $x$ -axis simultaneously and omission of the others, BQM can predict a rather empty interval with low intensity of particles that has a length

$$L \simeq 2\langle y \rangle \simeq \frac{\hbar t}{m\sigma_0^2} \langle y_0 \rangle, \quad (29)$$

if the constraint  $\Delta y \ll L$  is satisfied. The last constraint at  $\hbar t/2m\sigma_0^2 \gg 1$  condition, corresponds to  $\Delta y_0 \ll \langle y_0 \rangle$ . Therefore, it is shown that, according to BQM and under the conditions

$$\begin{aligned} \frac{\hbar t}{2m\sigma_0^2} &\gg 1, \\ Y &\ll \sigma_0 \ll \langle y_0 \rangle, \end{aligned} \quad (30)$$

a considerable position change in the  $y$ -coordinate of the source produces a region with very low intensity on the screen which is not predicted by SQM. Therefore, we have disagreement between the two theories' predictions even if  $\Delta y_0 \sim \sigma_0$  is considered as a constraint.

However, since the two particles in this experiment are emitted in an unentangled state, the results obtained maybe seem unbelievable. In this regard, Struyve [13] based on our aforementioned factorizable wave function (19), believes that the two independent particles of this experiment cannot produce different predictions for SQM and BQM. He argues that [13], the results of the experiment will not be altered if we emit the two particles simultaneously or emit only one particle at a time, because the two particles are totally independent.

Although we agree along with him that the results of this experiment are rather strange, we believe that discussion of this experiment can help to make the SQM and BQM disagreement more exciting. Hence, in the following, we substantiate our previous arguments about this experiment.

At first, we examine the applied condition  $Y \ll \sigma_0$ , in a double-slit experiment with two unentangled particles. One may argue that this condition is meaningless, because based on the specifications of the set-up,  $Y$  represents the distance between the center of each slit to the  $x$ -axis, and therefore, the minimum value of  $Y$  approaches  $\epsilon + \sigma_0$ , where  $\epsilon$  is considered very small and represents the length of plane that separates the two slits. But, this objection can be answered by considering the overlapping of each particle's two Gaussian wave functions which are generated at the two near slits. The overlapping causes that the peak of each Gaussian wave approaches more and more the  $x$ -axis. In addition, under this condition, the Gaussian wave functions lose their symmetrical form at each slit. Our argument becomes more clearer when we consider  $\epsilon = 0$  as a limiting case, i.e., we have only one slit. In this limiting case, it is clear that  $Y = 0$ . Therefore, when the two slits are very near together, the peak of Gaussian wave functions, i.e.  $Y$ , come very near to the  $x$ -axis and the condition  $Y \ll \sigma_0$  is completely right.

Another problem can be raised when one thinks of the two independent particles, as mentioned by Struyve too [13]. To handle to this problem, let us reconsider the second term in eq. (20) particularly the coefficient  $(\psi_{A_1}\psi_{A_2} - \psi_{B_1}\psi_{B_2})$ . Using the Gaussian wave (5) and the condition  $k_y \simeq 0$ , the latter coefficient can be written in the form

$$\begin{aligned} \psi_{A_1}\psi_{A_2} - \psi_{B_1}\psi_{B_2} = & (2\pi\sigma_t)^{-1/4} e^{2i(k_x x - E_x t/\hbar)} e^{-(y_1^2 + y_2^2)/4\sigma_0\sigma_t} e^{-(Y + u_y t)^2/2\sigma_0\sigma_t} \\ & \times [e^{(y_1 + y_2)(Y + u_y t)/2\sigma_0\sigma_t} - e^{-(y_1 + y_2)(Y + u_y t)/2\sigma_0\sigma_t}]. \end{aligned} \quad (31)$$

If we require that  $y_1(t) + y_2(t) = 0$ , i.e., the  $y$ -component of the two particles are entangled, then we would obtain the equation of motion (9), as expected. But our two particles in this experiment are initially unentangled and it is not necessary to have  $y_1(t) + y_2(t) = 0$ . Instead, we can have another selection on the two-slit set-up. In fact, if we apply the condition  $Y \ll \sigma_0$ , again the behavior of the equation of motion of the two particles in the  $y$ -direction is similar to the motion of the two entangled particles, while the two particles were unentangled. Hence, we can state that the classical interaction of the wave function of the two unentangled particles with the two-slit plane barrier for the condition  $Y \ll \sigma_0$ , results in a wave function which now guides the  $y$ -component of the center of mass of the two apparently unentangled particles in the same way as the case of two entangled particles with the initial condition  $-\sigma_0 \leq (y_1 + y_2)_{t=0} \leq \sigma_0$ , for those pairs of particles that pass through the two slits. Thus, we have shown that the results obtained in the two-slit experiment using two synchronized identical particles and the selective detection, are completely different from the ones obtained in a single-particle double-slit experiment, contrary to Struyve's belief [13]. In fact, the motion of either particle is now dependent on its own location and the location of the other particle, although the apparent form of the wave function of the system can be efficiently represented by the use of the unentangled form in (19), which is only useful at the ensemble level of particles. Therefore, our previous basic results about this experiment still remain intact.

### C. An experiment with two double slits and two entangled particles

The set-up of this experiment [6] consists of two double-slit screens where the slits  $A$  and  $B$  are on the right screen and  $A'$  and  $B'$  on the left screen, with their centers located at the points  $(\pm d, \pm Y)$  in a two-dimensional coordinate system. A special source which emits pairs of identical non-relativistic entangled particles is placed at the origin of the coordinates.

The entanglement property of the two particles is expressed by

$$\begin{aligned}x_1 + x_2 &= y_1 + y_2 = 0, \\ p_{1x} - p_{2x} &= p_{1y} - p_{2y} = 0.\end{aligned}\tag{32}$$

As we mentioned previously in subsection II. A, since we have

$$[(\hat{x}_1 + \hat{x}_2), (\hat{p}_{1x} - \hat{p}_{2x})] = [(\hat{y}_1 + \hat{y}_2), (\hat{p}_{1y} - \hat{p}_{2y})] = 0,\tag{33}$$

the joint properties  $(x_1 + x_2)$  and  $(p_{1x} - p_{2x})$  as well as  $(y_1 + y_2)$  and  $(p_{1y} - p_{2y})$  can both be determined with the desired accuracy. Thus, for example, determination of  $y_0 = \frac{1}{2}(y_1 + y_2)_{t=0}$  is theoretically possible. Since the source used in this experiment is the same as the one used in the EPR experiment [14], it seems that performing this experiment can be considered more feasible than those of the last two aforementioned experiments. In addition, such sources can be utilized in some interesting processes of the quantum information theory [12] such as quantum dense coding and quantum teleportation protocols [15].

The general form of the wave function for this system can be written as

$$\begin{aligned}\psi(x_1, y_1; x_2, y_2; t) &= \tilde{N} [\psi_A(x_1, y_1, t) \psi_{B'}(x_2, y_2, t) \pm \psi_A(x_2, y_2, t) \psi_{B'}(x_1, y_1, t) \\ &\quad + \psi_B(x_1, y_1, t) \psi_{A'}(x_2, y_2, t) \pm \psi_B(x_2, y_2, t) \psi_{A'}(x_1, y_1, t)].\end{aligned}\tag{34}$$

Using BQM, it is straightforward to show that eq. (9) again determines the motion of the center of mass of the  $y$ -coordinate of the two entangled particles.

Once again, we can have a similar discussion on SQM's probabilistic asymmetrical prediction against BQM's deterministic and symmetrical prediction, with the  $y_0 = 0$  condition. In addition, for the conditions

$$\begin{aligned}\frac{\hbar t}{2m\sigma_0^2} &\sim 1, \\ Y &\sim \sigma_0, \\ 0 \leq \Delta y_0 &\ll \sigma_0,\end{aligned}\tag{35}$$

one can again obtain a reasonable symmetrical prediction, based on BQM. It is worthy to note that, the first two conditions provide a considerable overlap of the wave packets on the screen, so that SQM cannot predict a symmetrical detection. Therefore, our all previous conclusions about this experiment are unchanged as well.

### III. CONCLUSION

Three recent proposed experiments have been studied in some details. It is shown that, the objections raised by Marchildon [7,9] as well as Struyve and De Baere [8,13] are not justified and that the basic conclusion of the experiments, that is, the existence of the incompatibility between SQM's probabilistic prediction and BQM's deterministic prediction, still stands out. In fact, they tried to show equivalence of the two theories at the individual level of the suggested experiments, by applying SQM's rules to both theories. For the present experimental ability the only difficulty seems to be related to the special properties of the sources which produces the pairs of entangled fermionic particles, such as electrons, neutrons or entangled bosonic particles. However, it seems that such experiments may be done using photons at Pavia, as Ghose promised in [10,11]. Furthermore, a new and more feasible experimental set-up to distinguish between SQM and BQM has been suggested elsewhere [16] which will make the discussion on the disagreement between the two theories more serious and more interesting than what has been done so far.

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